TRACKING CODE DEVELOPEMENTS

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BENE meeting

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TOPIC :

DEVELOPEMENTS OF NUMERICAL METHODS FOR A 3-D SIMULATION OF A FFAG RADIAL SECTOR MAGNET : A NUMERICAL INTERPOLATION AND A ANALYTICAL CALCULATION.

THE RAY TRACING METHOD

Integration of Lorentz equation based on Taylor series (the choice of the order will be discuss in the next slides) :

$$\bullet Position \ : \ \vec{R}(M_1) \approx \vec{R}(M_0) + \vec{u}(M_0)\,\Delta s + \vec{u}\,'(M_0)\,\frac{\Delta s^2}{2!} + ... + \vec{u}\,'''''(M_0)\,\frac{\Delta s^6}{6!} \ \ [+...] \qquad (1)$$

$$\bullet Velocity \ : \ \vec{u}(M_1) \approx \vec{u}(M_0) + \vec{u}\,'(M_0)\,\Delta s + \vec{u}\,''(M_0)\,\frac{\Delta s^2}{2!} + ... + \vec{u}\,'''''(M_0)\,\frac{\Delta s^5}{5!} \ \ [+...]$$

using $\vec{u}\,' = \vec{u} \times \vec{B}$ (Lorentz equation), $\vec{u}\,'' = \vec{u}\,' \times \vec{B} + \vec{u} \times \vec{B}\,'$, $\vec{u}\,''' = ...$ *etc.*

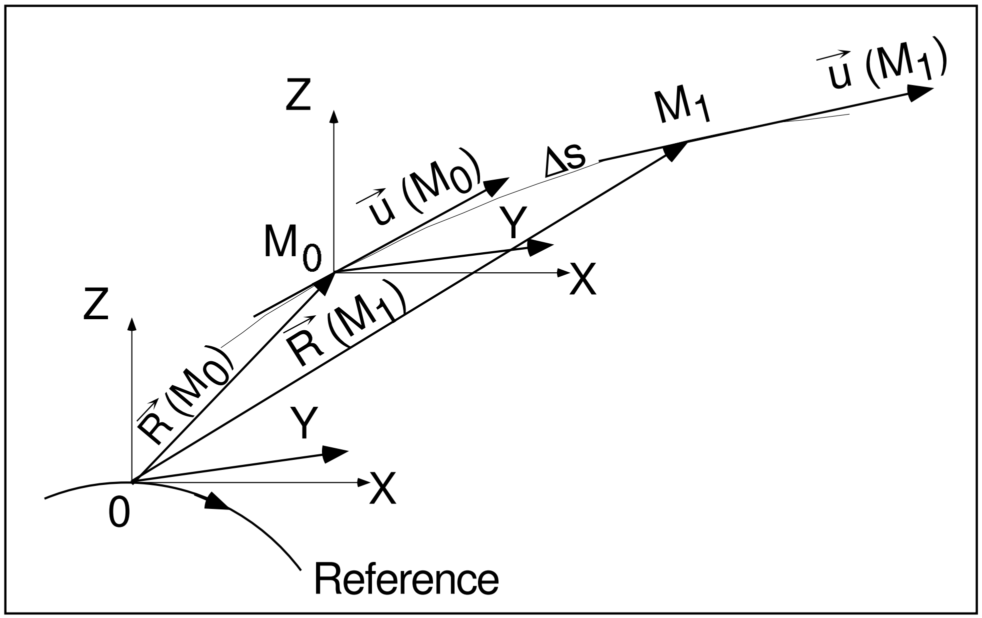


Fig. 1 – Position and velocity of a particle in the reference frame.

THE FOUR STEPS TO CALCULATE $\vec{B}(s)$ AND $d^n\vec{B}/ds^n$

1. Sector mid-plane field model of the form : $B_z(r,\theta) = B_{z0}\,\mathcal{F}(r,\theta)\,\mathcal{R}(r)$, yielding $\partial^{i+j}B_z/\partial\theta^i\partial r^j$

   $\mathcal{F}(r,\theta)$: field fall-offs coefficient

   $\mathcal{R}(r,\theta) = (r/R_0)^K$ for a FFAG magnet

   $\mathcal{R}(r,\theta) = b_0 + b_1\,(\frac{r-R_0}{R_0}) + b_2\,(\frac{r-R_0}{R_0})^2 + ... + b_5\,(\frac{r-R_0}{R_0})^5$ for a DIPOLE magnet

2. Next, transform from magnet cylindrical frame into Zgoubi Cartesian frame, using

   $\partial B_z/\partial X = (1/r)\partial B_z/\partial\theta,\;\; \partial B_z/\partial Y = \partial B_z/\partial r,\;\; \partial^2 B_z/\partial X^2 = (1/r^2)\partial^2 B_z/\partial\theta^2 + (1/r)\partial B_z/\partial r,$

3. Z-derivatives and extrapolation off mid-plane yield yield the 3-D $\vec{B}$ model

   $$\boxed{\vec{B}(X,Y,Z), \qquad \partial^{i+j+k}\vec{B}/\partial X^i\partial Y^j\partial Z^k}$$

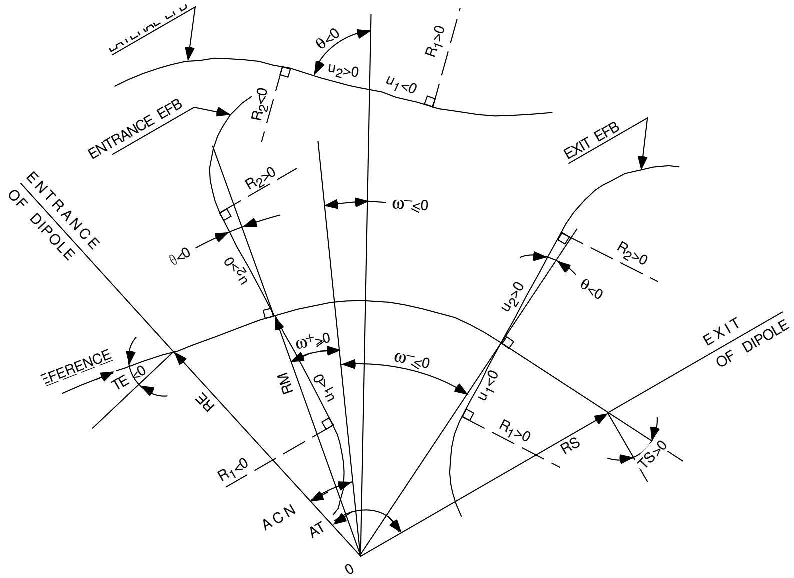
4. Eventually, the derivatives $\boxed{d^n\vec{B}/ds^n}$ needed in Eqs. 1 are derived from the above using

   $$\vec{B}'(s) = \sum_i \frac{\partial\vec{B}(X,Y,Z)}{\partial X_i}\,u_i(s)\,, \qquad \vec{B}''(s) = \sum_{ij}\frac{\partial^2\vec{B}(X,Y,Z)}{\partial X_i\partial X_j}\,u_i(s)\,u_j(s) + \sum_i \frac{\partial\vec{B}(X,Y,Z)}{\partial X_i}\,u_i'(s$$

   $(X_{i,j,...},\;\; i,j,... = 1,3$ stand for $X,\;Y$ or $Z)$.

AN N-UPLET MAGNET PROCEDURE

GEOMETRY/FIELD FOR ONE DIPOLE :



When alone, a dipole is encompassed in the magnetic field region defined by the angle AT. The dipole geometrical parameters yield the mid-plane vertical field :

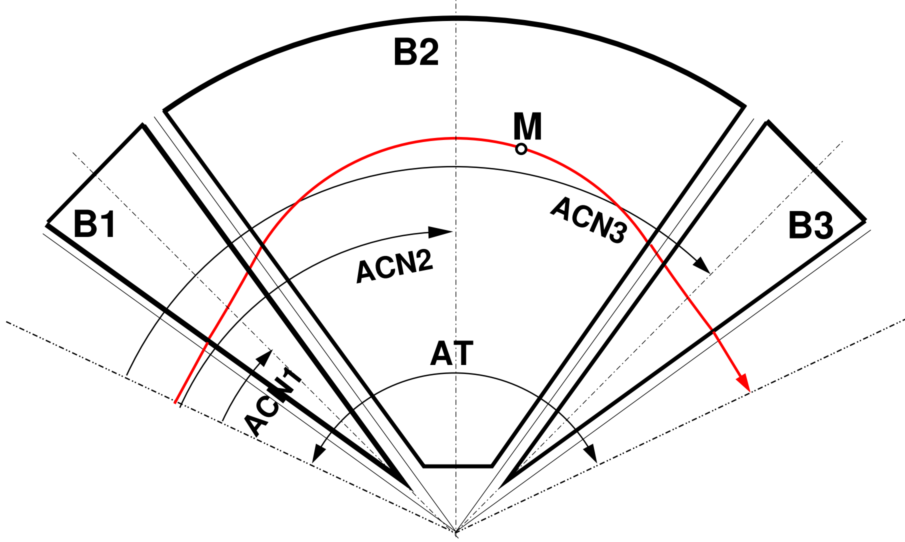
$$B_{zi}(r,\theta) = B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r)$$

by using

$$\mathcal{F}_i(r,\theta) = \mathcal{F}_{\text{Entrance}}(r,\theta)\times\mathcal{F}_{\text{Exit}}(r,\theta)\times\mathcal{F}_{\text{Lateral}}($$

simulating the effect of EFB's $\longrightarrow$ discussed next slide.

GEOMETRY/FIELD FOR A SECTOR TRIPLET :



Basically, N=3 (here) dipoles are encompassed in the magnetic field region defined by the angle AT, they are positioned by $ACNi$. Obtaining the total field is a matter of summation over the N neighboring dipoles (not more than a trick)

$$B_z(r,\theta) = \sum_{i=1}^{N} B_{zi}(r,\theta) = \sum_{i=1}^{N} B_{z0,i}\,\mathcal{F}_i(r,\theta)\,\mathcal{R}_i(r) \tag{2}$$

$$\text{Derivatives :}\ \frac{\partial^{i+j}\vec{B}_z(r,\theta)}{\partial\theta^i\partial r^j} = \sum_{i=1,N} \frac{\partial^{i+j}\vec{B}_{zi}(r,\theta)}{\partial\theta^i\partial r^j} \tag{3}$$

THE FIELD FALL-OFFS SIMULATION

Field fall-off at an *EFB* (*Entrance*, *Exit*) is modeled by the Enge model :

$$\mathcal{F}_{EFB}(d) = \frac{1}{1+\exp[P(d)]}\ , \qquad P(d) = C_0 + C_1\frac{d}{g} + C_2\left(\frac{d}{g}\right)^2 + \ ... \ + C_5\left(\frac{d}{g}\right)^5$$

– $d(r,\theta)$ is the distance to the *EFB*

– Possible gap variation is accounted for g homogeneous to the gap, e.g :

$g(r) = g_0(R_0/r)^K$ (pole shaping),
or
$g = C^{\underline{te}}$ (coil shaping)

– coefficients $C_0 - C_5$ determine the shape of the field fall-off (can be determined from prior POISSON or TOSCA calculations for instance)

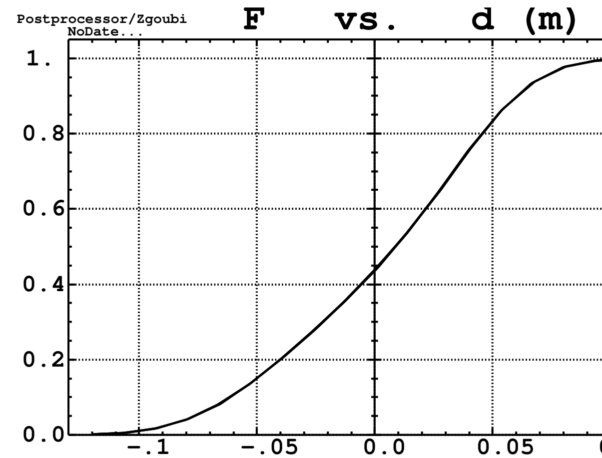


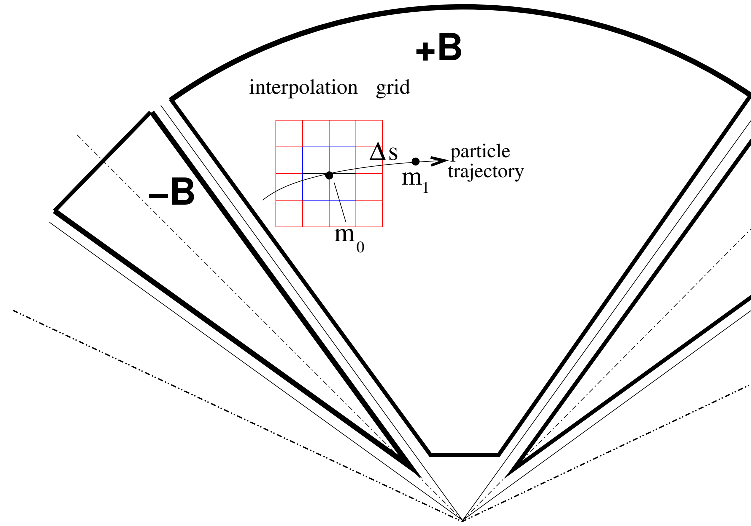
Fig. 2 – Fringe field $\mathcal{F}_{EFB}$ as a function of distance d (gap $g = 8$ cm, $C_0 = 0.1455$, $C_1 = 2.2670$, $C_2 = -0.6395$, $C_3 = 1.1558$, $C_4 = C_5 = 0$).

CALCULATION OF FIELD DERIVATIVES : TWO METHODS

Numerical interpolation using a 'flying' mesh : 2nd order 3 x 3 nodes or 4th order : 5 x 5 nodes

$$B(r,\theta) = A_{00} + A_{10}\theta + A_{01}r + A_{20}\theta^2 + A_{11}\theta r + A_{02}r^2 + \ldots.. + A_{22}\theta^2 r^2 + A_{13}\theta r^3 + A_{04}r^4$$

The source code contains the analytical expressions of the coefficients $A_{kl} = \frac{1}{k!l!}\frac{\partial^{k+l}B(r,\theta)}{\partial\theta^k\partial r^l}$

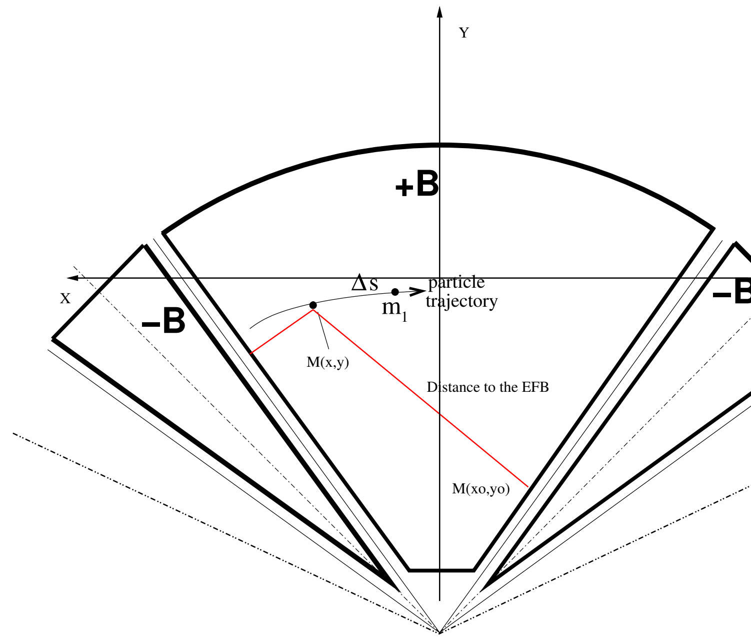


Analytical Calculation :

$$x(r,\theta) = \cos(ACN - \theta) - R_0, \quad \text{etc ..}$$

$$d(r,\theta) = \sqrt{(x(r,\theta) - x_0(r,\theta))^2 + (y(r,\theta) - y_0(r,\theta))^2}$$

It yields $\frac{\partial^{u+v}d(r,\theta)}{\partial\theta^u\partial r^v}$, $\frac{\partial^{u+v}F(r,\theta)}{\partial\theta^u\partial r^v}$, $\frac{\partial^{u+v}P(r,\theta)}{\partial\theta^u\partial r^v}$ and finally $\frac{\partial^{u+v}B(r,\theta)}{\partial\theta^u\partial r^v}$



COMPARAISON ORDER 2 VS ORDER 4

To test if it is necessary to compute the derivatives up to the fourth order in term of calculation accuracy, we make tracking (2000 turns) on a stable limit orbit and check the numbers of turns the particle did before to be lost and that for the two methods (interpolation/Analytic) and different steps of integration.

We did that test for two kinds of triplet used to simulate the **150 Mev proton ring triplet**.

- The first one with $K_D = 7.25$ and $K_F = 7.58$ (used for the first simulations)
- The second one with $K_D = K_F = 7.6$ (new triplet more realistic, horizontal tunes perfectly constant)

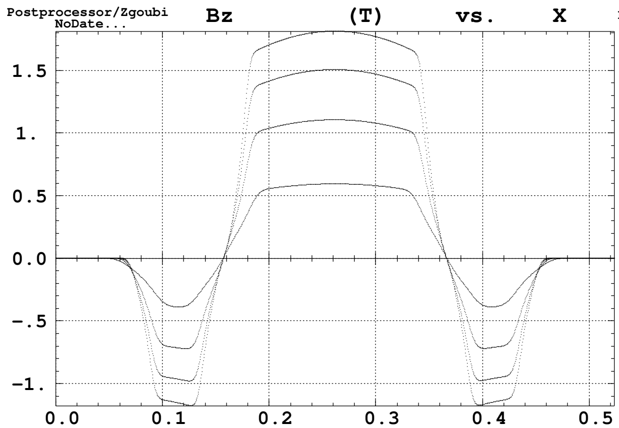


FIG. 3 – $K_D = 7.25$ and $K_F = 7.58$, closed orbit E = 12,50,100,150 Mev

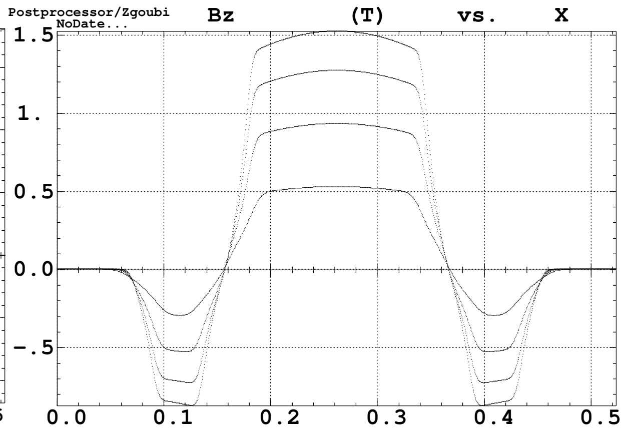


FIG. 4 – $K_D = K_F = 7.6$ E = 12,43,85,125 Mev

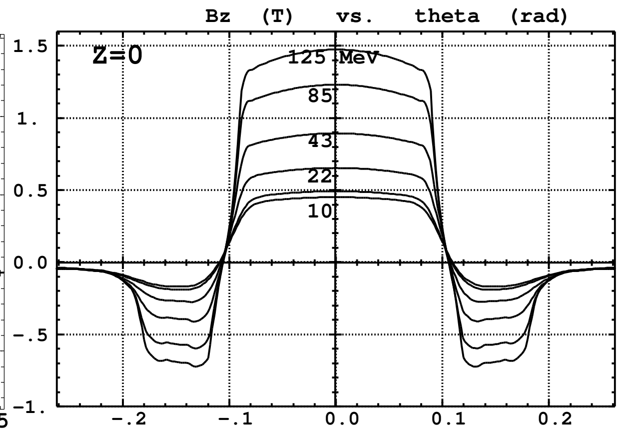


FIG. 5 – Fiel compute from Tosca 3-D map E = 12,43,85,125 Mev

SAMPLE PHASE-SPACE MOTION (2000 TURNS) : TRIPLET $K_D = 7.25$, $K_F = 7.58$ E=50 Mev, $r = r_{co}$, z=2 cm

2nd order $\delta s = 0.25cm$: Numeric (left), Analytic(right)

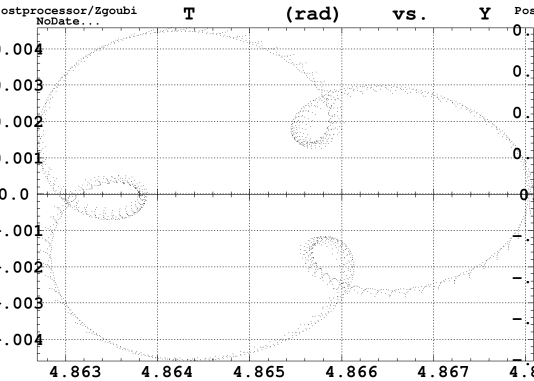


FIG. 6 – horizontal motion

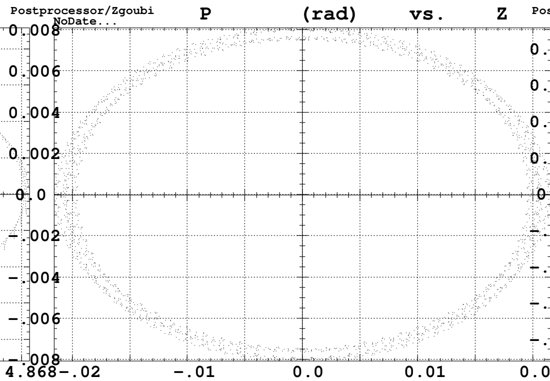


FIG. 7 – vertical motion

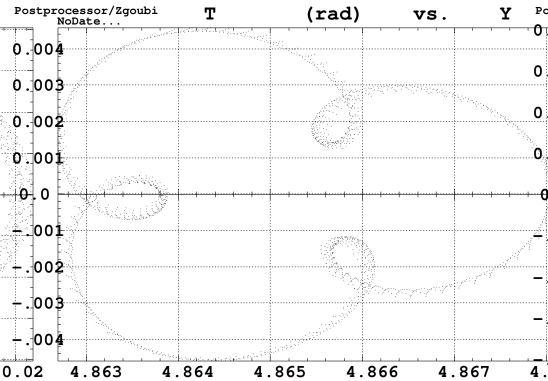


FIG. 8 – horizontal motion

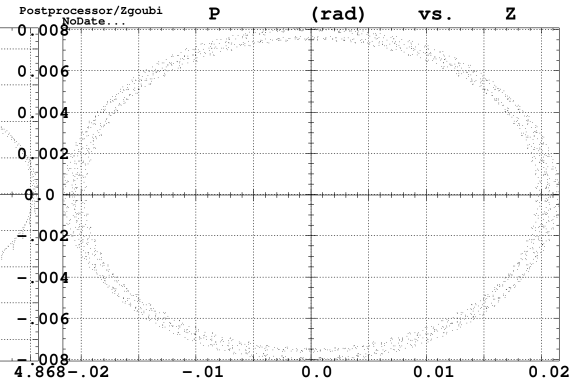


FIG. 9 – vertical motion

4th order $\delta s = 0.25cm$: Numeric (left), Analytic(right)

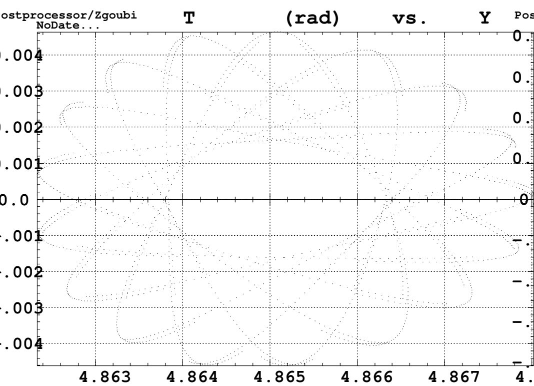


FIG. 10 – horizontal motion

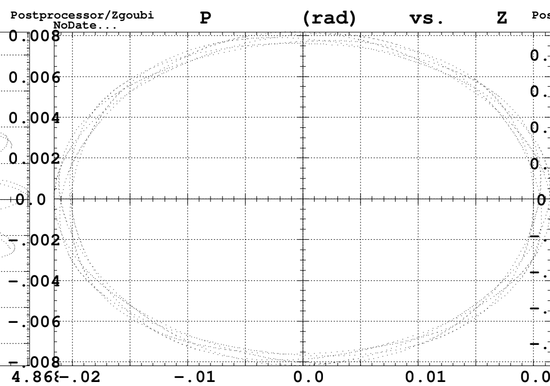


FIG. 11 – vertical motion

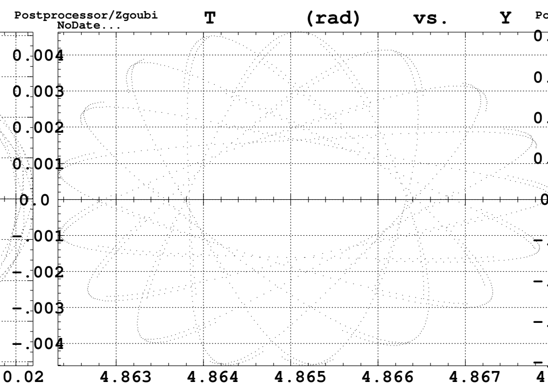


FIG. 12 – horizontal motion

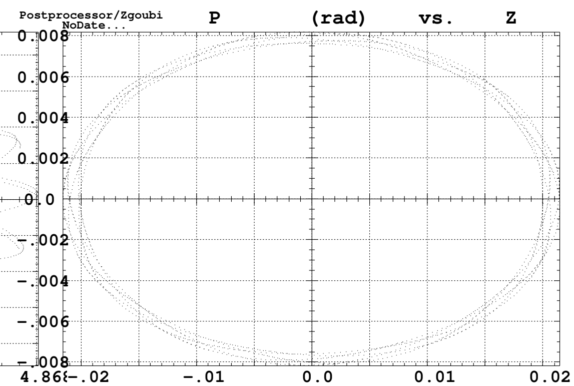


FIG. 13 – vertical motion

SAMPLE PHASE-SPACE MOTION (2000 TURNS) : TRIPLET $K_D = K_F = 7.6$, E=43 Mev, $r = r_{co}$ z=2 cm

2nd order $\delta s = 0.25cm$: Numeric (left), Analytic(right)

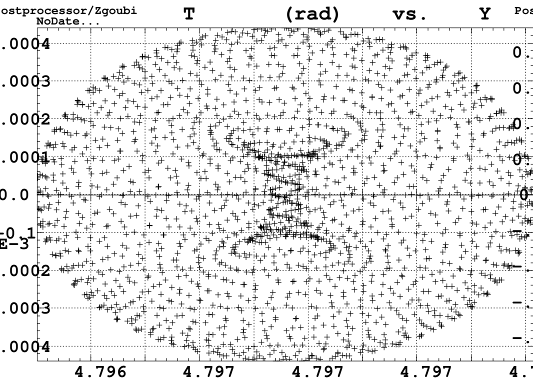


FIG. 14 – horizontal motion

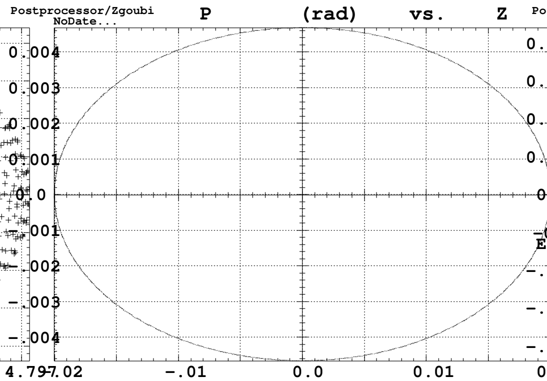


FIG. 15 – vertical motion

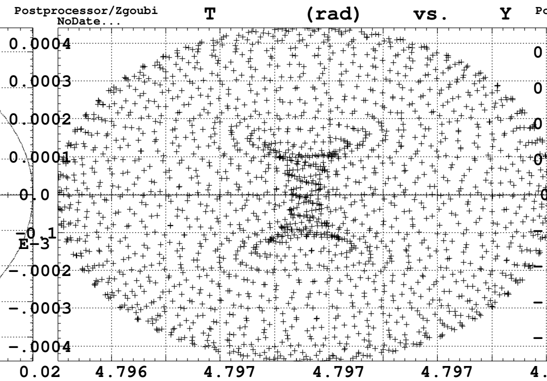


FIG. 16 – horizontal motion

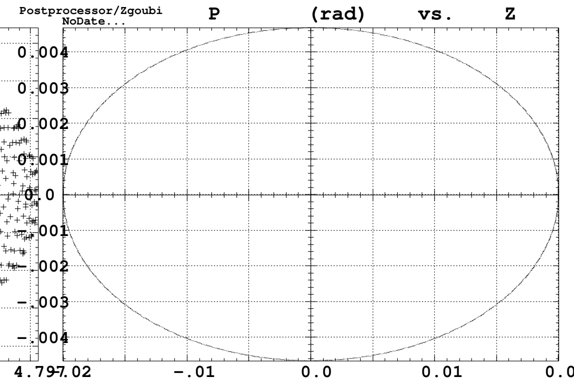


FIG. 17 – vertical motion

4th order $\delta s = 0.25cm$: Numeric (left), Analytic(right)

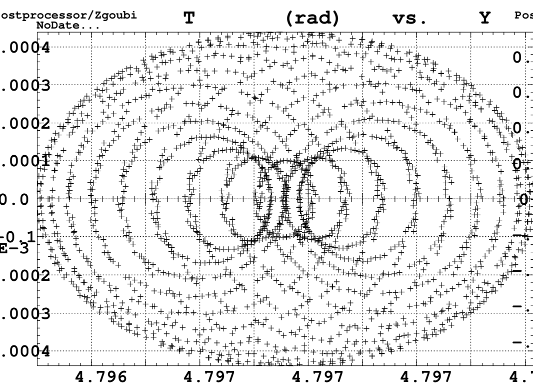


FIG. 18 – horizontal motion

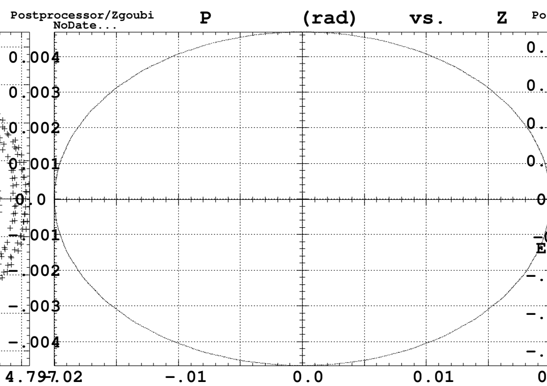


FIG. 19 – vertical motion

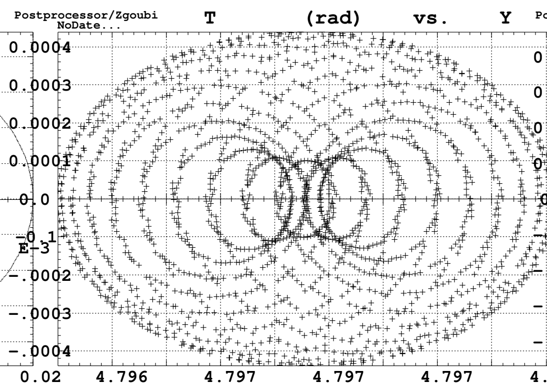


FIG. 20 – horizontal motion

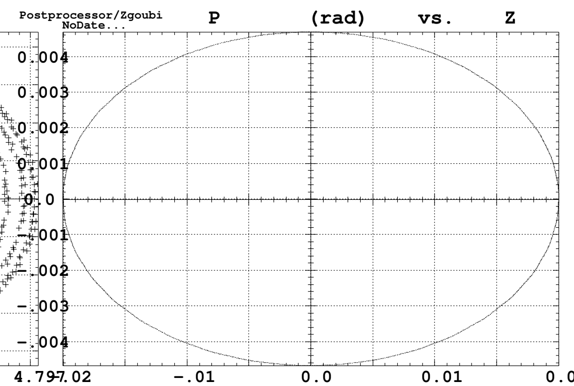


FIG. 21 – vertical motion

RESULTS

For the first triplet ($K_D = 7.25$, $K_F = 7.58$), the results for the stable limits orbits (E =100 Mev ,$r = r_{co} + 1.4cm$, z=0.4cm, E =50 Mev ,$r = r_{co} + 1.9cm$, z=0.7 cm) and show that for the 2nd order it takes less and less turn for the particle to be lost when the step integration increases. For the 4th order the number of turn varies with the step size but doesn't follow a specific law.

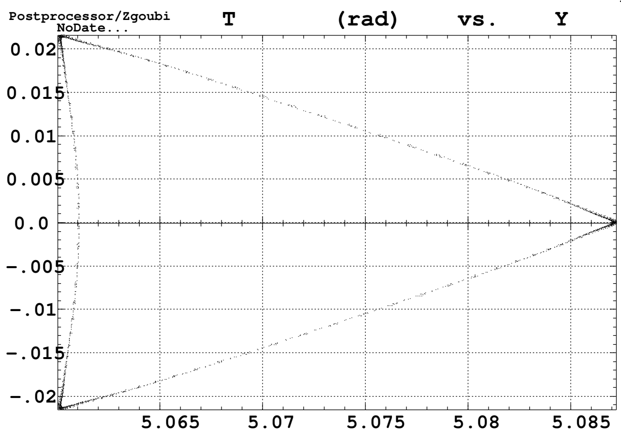


FIG. 22 – Motion on the stable limit : step size = 0.25 cm, analytic calculation, 4th order

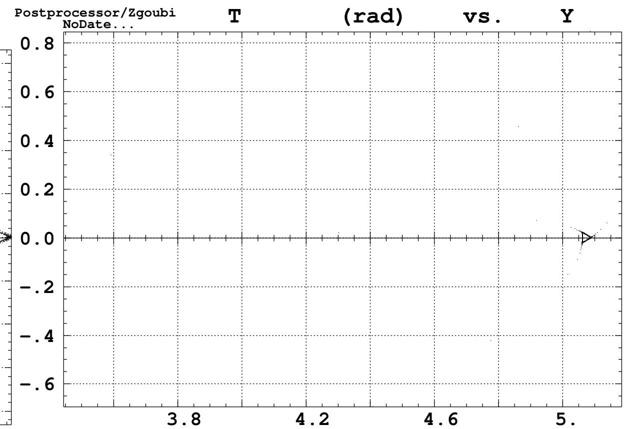


FIG. 23 – Motion on the stable limit , the particle is lost after 966 turns :step size = 0.25 cm interpolation calculation, 2nd order

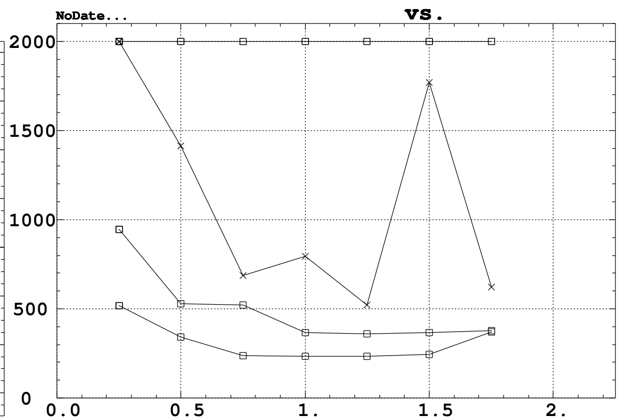


FIG. 24 – Number of turns on the stable limits orbits : E =100 Mev ,$r = r_{co} + 1.4cm$ and E =50 Mev ,$r = r_{co} + 1.9cm$, z=0.7cm vs the step integration size

For the second triplet ($K_D = K_F = 7.6$) the maximum number of turns, a particle stays on a stable limit, is never more than 20 or 30 and it is **the same for the 2nd or the fourth order and for analytic or interpolation calculation.**

CONCLUSION

1. For a triplet with $K_D \neq K_F$ the accuracy of calculation depends on the order 2nd or 4th. If we decide to make tracking faster using the 2nd order we take the risk to loose some particles because of the worse accuracy of the calculation.

2. For a more realistic triplet with $K_D = K_F$ the accuracy seems to not depend on the order of calculation we choose to use.
   The analytic calculation is about 2.3 faster than the interpolation one and the 4th order is about 2.4 slower than the 2nd one. **That means that a 2nd order analitycal tracking in that case is 5,5 faster then a 4th order interpolation one.**