Tracking in KEK 150 MeV FFAG rings and others

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Main topic :

3-D simulation of the field in an FFAG radial sector magnet, from its geometrical parameters. Application to multiturn tracking.

The method is derived from the simulation of large dipoles, as implemented in the 70's in the ray-tracing code Zgoubi for the design of large acceptance spectrometers

Motivations of this FFAG magnet simulation :

- **develop an efficient ray-tracing tool for further NuFact FFAG studies - design, tracking**
- **provide field simulation from purely analytical expressions of 3-D field - no field maps, no interpolation !**
- **yield fast, high precision tracking**
- **allow fast optimization of magnet geometry as constrained by machine parameters, using for instance automatic matching procedures**

Contents

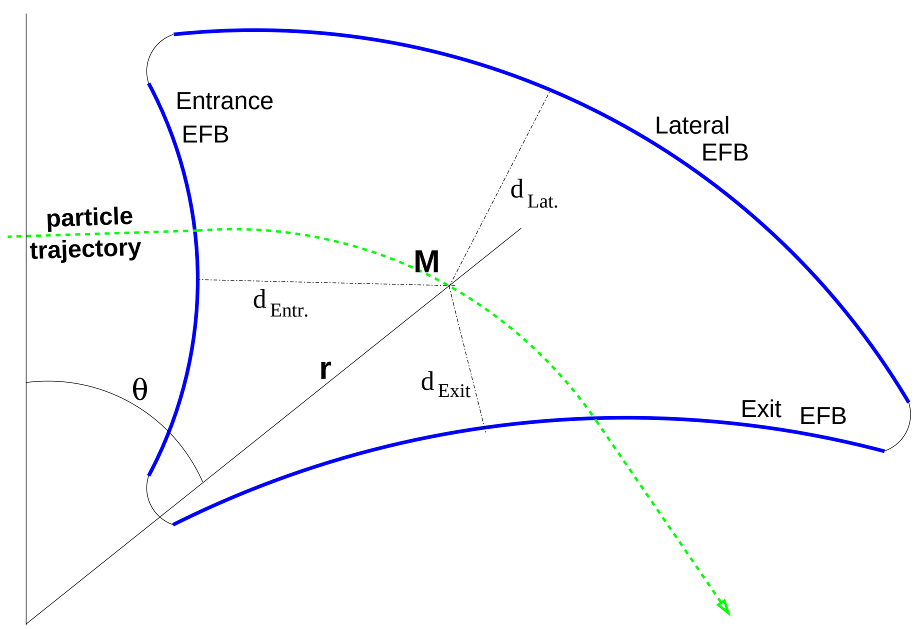
1 A former "*DIPOLE*" procedure

... the FFAG one will be derived from : details on that in F. Lemuet's talk, this meeting.

An analytical dipole mid-plane model $B_z(r,\theta)$ is provided in Zgoubi by the *DIPOLE* procedure,
- installed in the code in the early 70's for the design of SPES2 spectrometer at SATURNE (Saclay),
- used since then for the design of several large spectrometers :
SPES3 and others at SATURNE, SPEG (GANIL, Caen), Kaon-QD (GSI), etc.

A regular way of simulating magnetic fields within the limits of "Effective Field Boundaries"(EFB)
- also used in other spectrometer codes (e.g., RAYTRACE, S. Kowalski) :

GEOMETRY OF A DIPOLE :



The field at position $M(r,\theta)$ on the trajectory is calculated from the distance of the surrounding EFB's :
1. each EFB is responsible of a field form factor that describes its field fall-off
2. the resulting form factor at M is the product of the individual form factors :

$$\mathcal{F}_i(r,\theta) = \mathcal{F}_{\textbf{Entrance}}(r,\theta) \times \mathcal{F}_{\textbf{Exit}}(r,\theta) \times \mathcal{F}_{\textbf{Lateral}}(r,\theta)$$

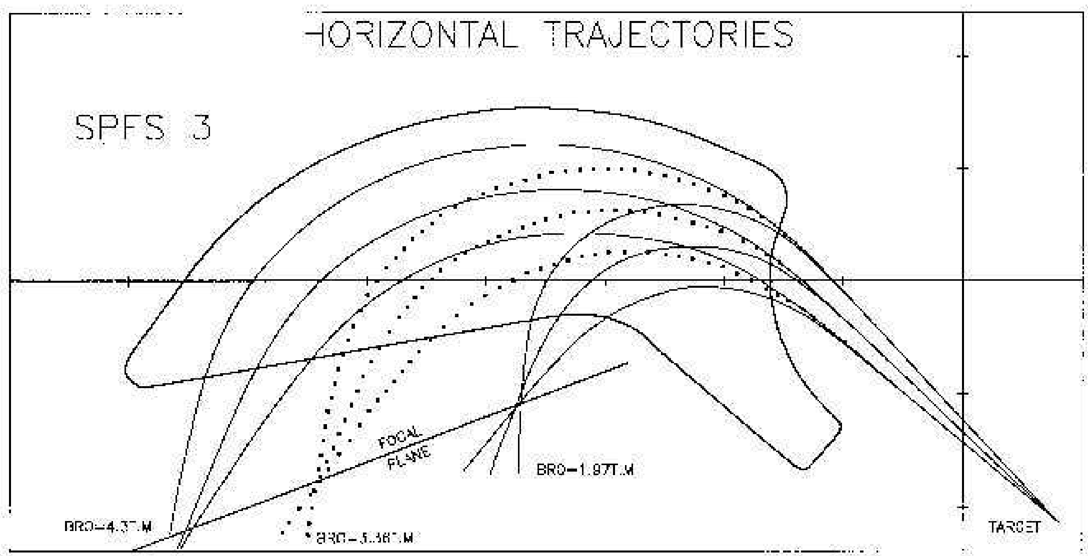
That yields a vertical field at M of the form

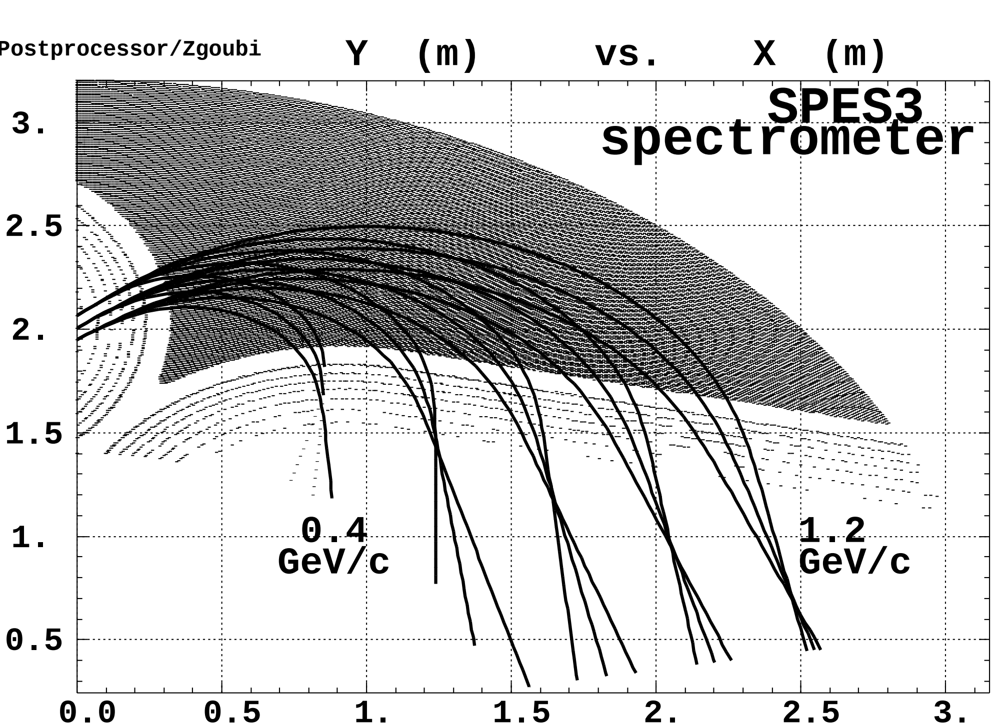
$$B_z(r,\theta) = B_{z0}\,\mathcal{F}(r,\theta)\,\mathcal{R}(r)$$

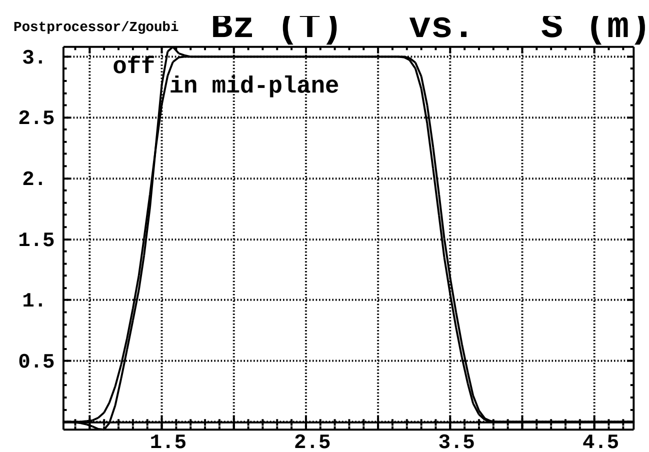
wherein $\mathcal{R}(r)$ is a possible radial dependence of B_z.

Example : Design of Elbeck spectrometer SPES3 using *DIPOLE*.

SPES3 is now at KEK!



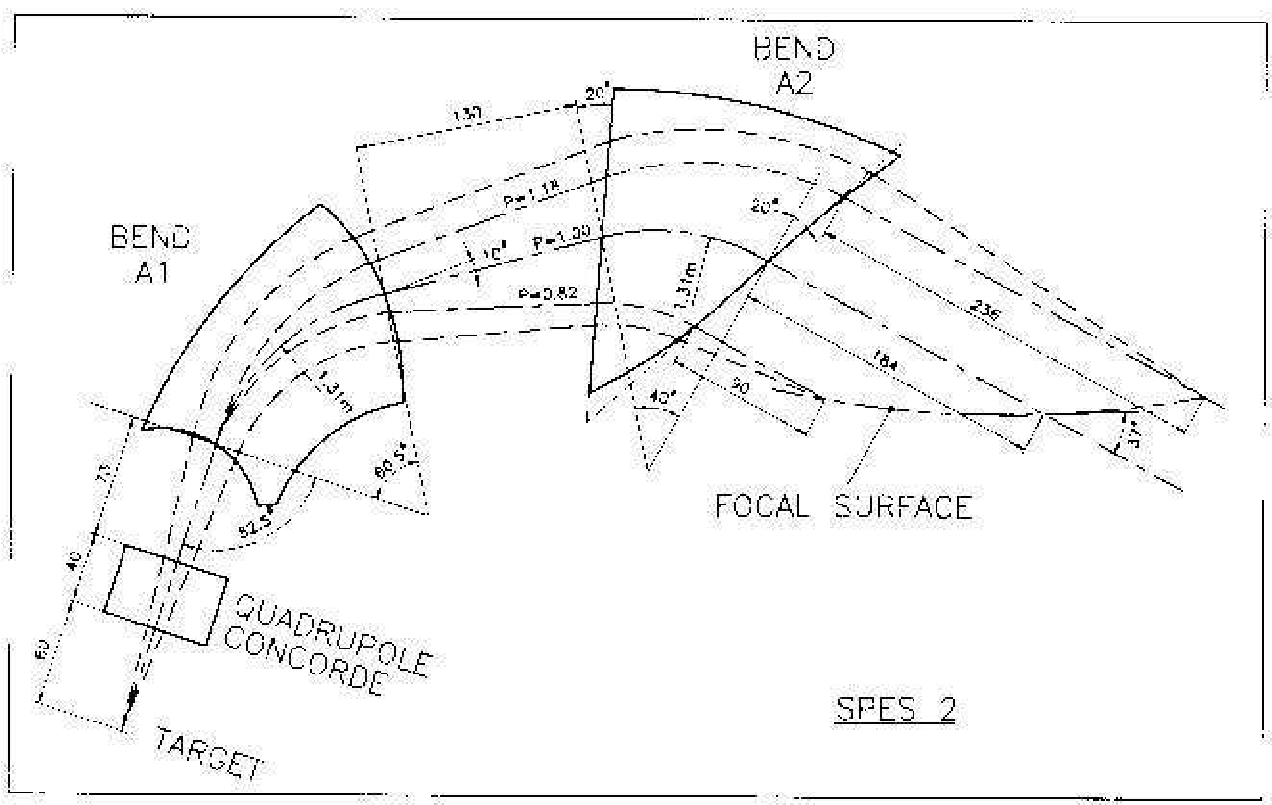




The construction of SPES3 used the geometrical parameters so optimized by means of "*DIPOLE*" (field form factors determined using POISSON) - no 3-D magnet simulations at that time.

Ex. #2 : Design of SPES2 using *DIPOLE*.

SPES2 is also at KEK!



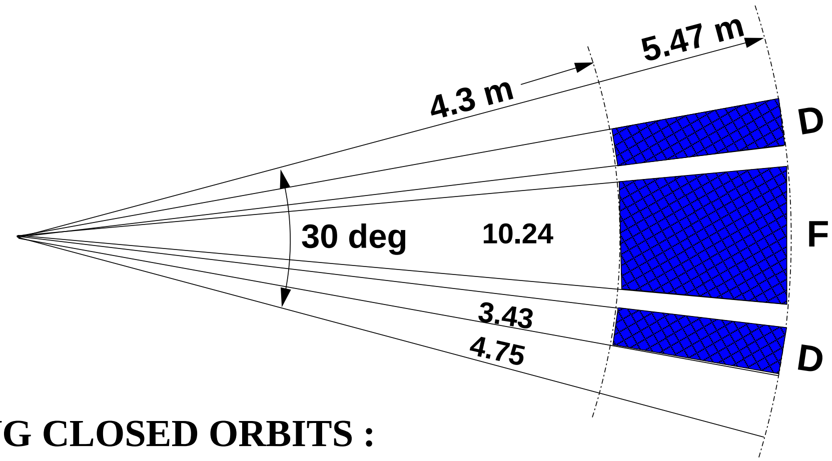

```
***  SPES2  600MEV/C
 'OBJET'
 2335.
 5
 .104113721 .832545509 0.104 1.85043641 0. .001
 0. 0. 0. 0. 0. 1.
 'ESL'
 60.
 'QUADRUPO'
  0
 40.  10. -5.61
 0.  0.
 6  .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
 0.   0.
 6  .1122 6.2671 -1.4982 3.5882 -2.1209 1.723
 3.
 1 0. 0. 0.
 'ESL'
 27.893
 'DIPOLE'           BEND A1
 2 0 0
 350  200
 15.273   0. 0. 0.
 107.  50.   131.  70.  180.
 46. -1.
 4   .14552 5.21405 -3.38307 14.0629 0. 0. 0.
  31.  17.5    -57.24  0.    0.  -57.24
 46. -1.
 4   .14552 5.21405 -3.38307 14.0629 0. 0. 0.
 -32. -10.    -158.    0.    0. -126.
 0
 25
 .3
 2
  138.548   -.33161256  144.648 .43827
 'ESL'
 24.21
 'DIPOLE'           BEND A2
 2 0 0
 350  200
 15.2730  0. 0. 0.
 78.   39.   131.  60.  210.
 46. -1.
 4   .14552 5.21405 -3.38307 14.0629 0. 0. 0.
  20.  20.     1.E6   -1.E6    1.E6   1.E6
 46. -1.
 4  .14552  5.21405  -3.38307  14.0629 0. 0. 0.
 -20.  20.    -350.  -26.5  20.5  800.
 0
 25
 .3
 2
  138.548   -.331612558   138.716 .33451
 'END'
```

In the case of the two SPES2 large acceptance dipoles "A1" and "A3", the optimization of the geometrical parameters was also accomplished using "*DIPOLE*". Their behavior was fully satisfying - the spectrometer did work fine !

2 Application : the KEK 150 MeV proton FFAG ring

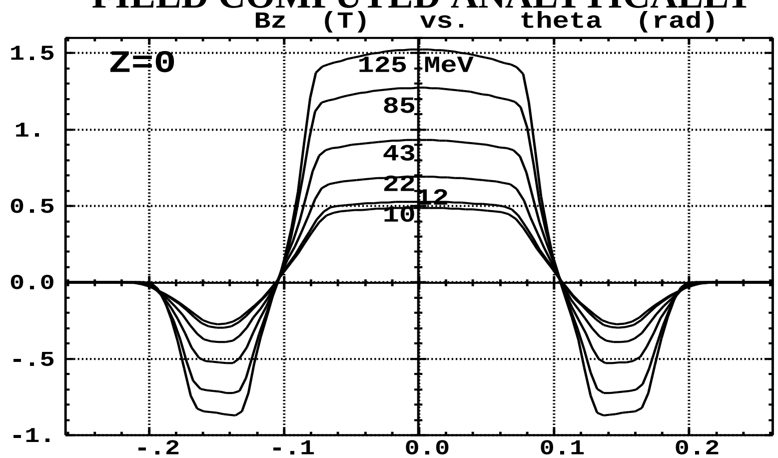
Figure 1: 150 MeV FFAG ring and of the injection cyclotron and 12 MeV line. The 150 MeV FFAG DFD triplet.

Goal : try to show that the method works fine...

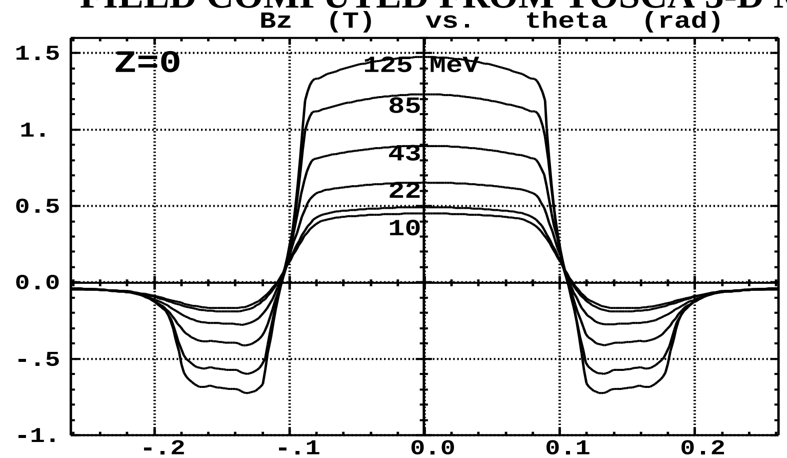


MID-PLANE FIELD ALONG CLOSED ORBITS :

FIELD COMPUTED ANALYTICALLY



FIELD COMPUTED FROM TOSCA 3-D MAP



The field in the case of the geometrical method has been tuned so as to yield results as close as possible to the TOSCA map case. This was done using two constraints :

1/ closed orbit must be the same as in the TOSCA map case at 43 MeV (value chosen because it is about half-way between r_{min} and r_{max}),

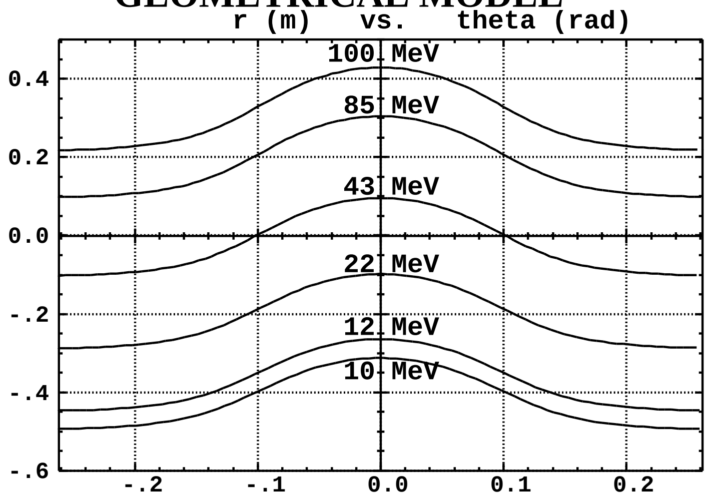
2/ same horizontal and vertical tunes with both methods.

• Condition 1/ yields BF, and condition 2/ yields BF/BD ratio. •

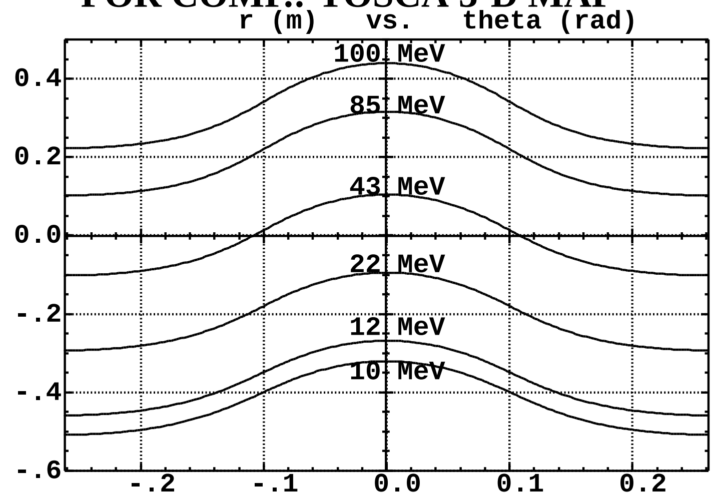
• *Remark :* no effort was made to have the analytical field shape closer to TOSCA case in the BD and drift regions. We are after that now!.

CLOSED ORBITS IN CELL :

GEOMETRICAL MODEL



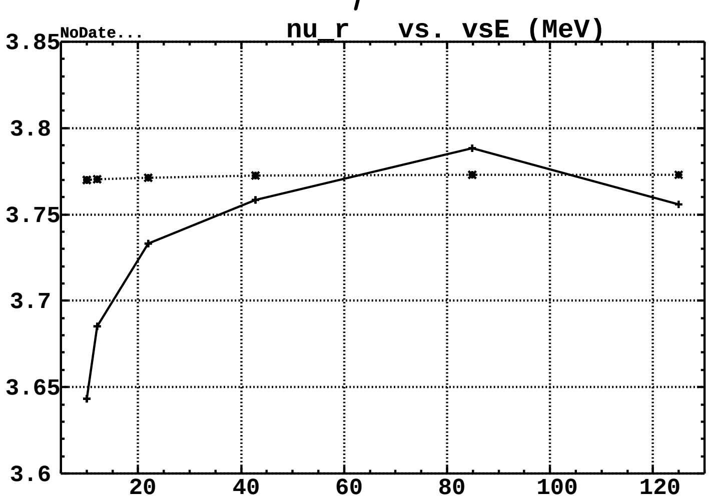
FOR COMP.: TOSCA 3-D MAP



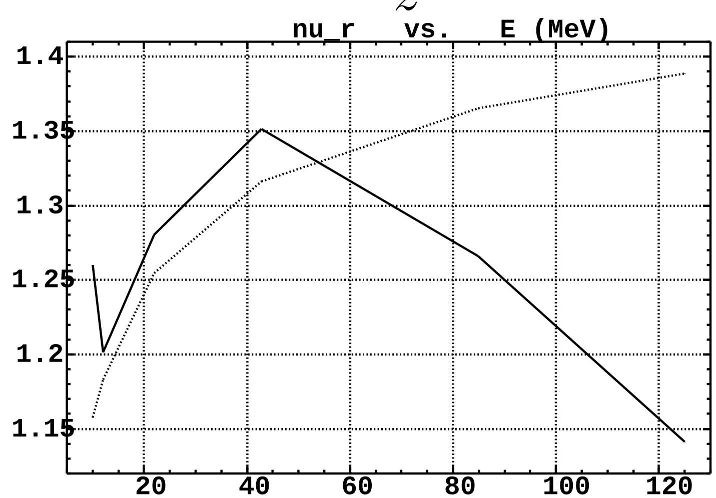
One observes a slightly larger overall excursion in the TOSCA map case, a kind of effect that could be obtained with a K value slightly smaller than 7.6 (that of the geometrical model).

TUNES (full turn, 12 cells) :

ν_r



ν_z



Machine tunes. Dashed lines are from the geometrical method, solid lines are from TOSCA map.

Tunes, comparison with RK4

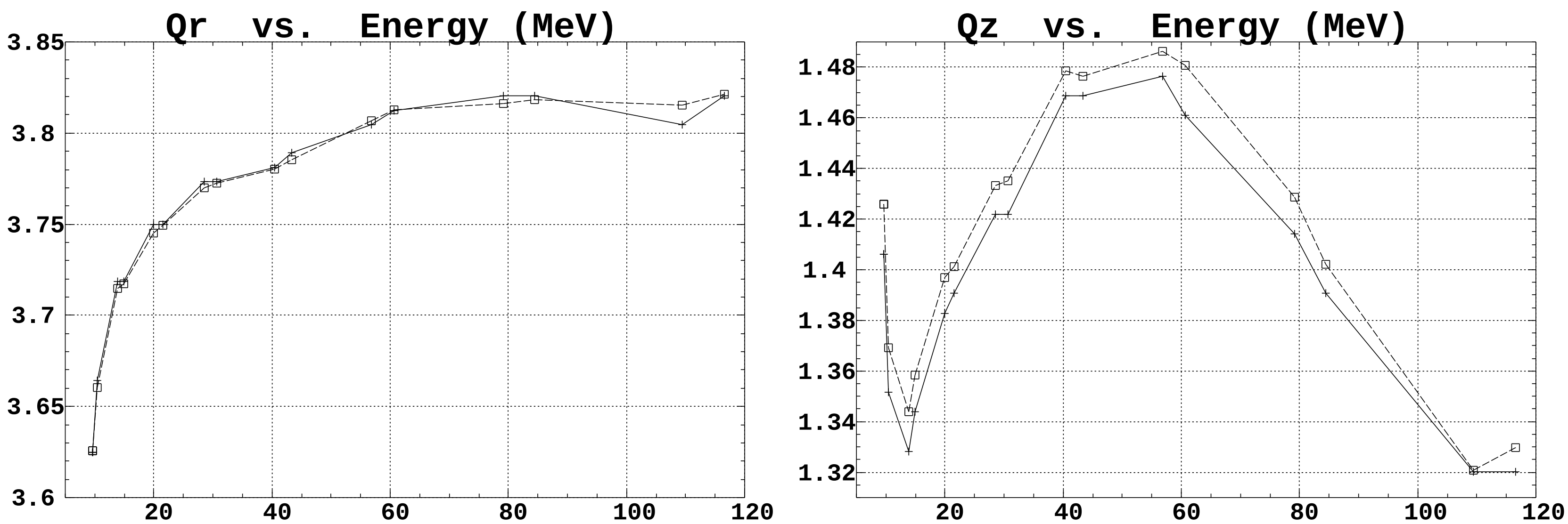
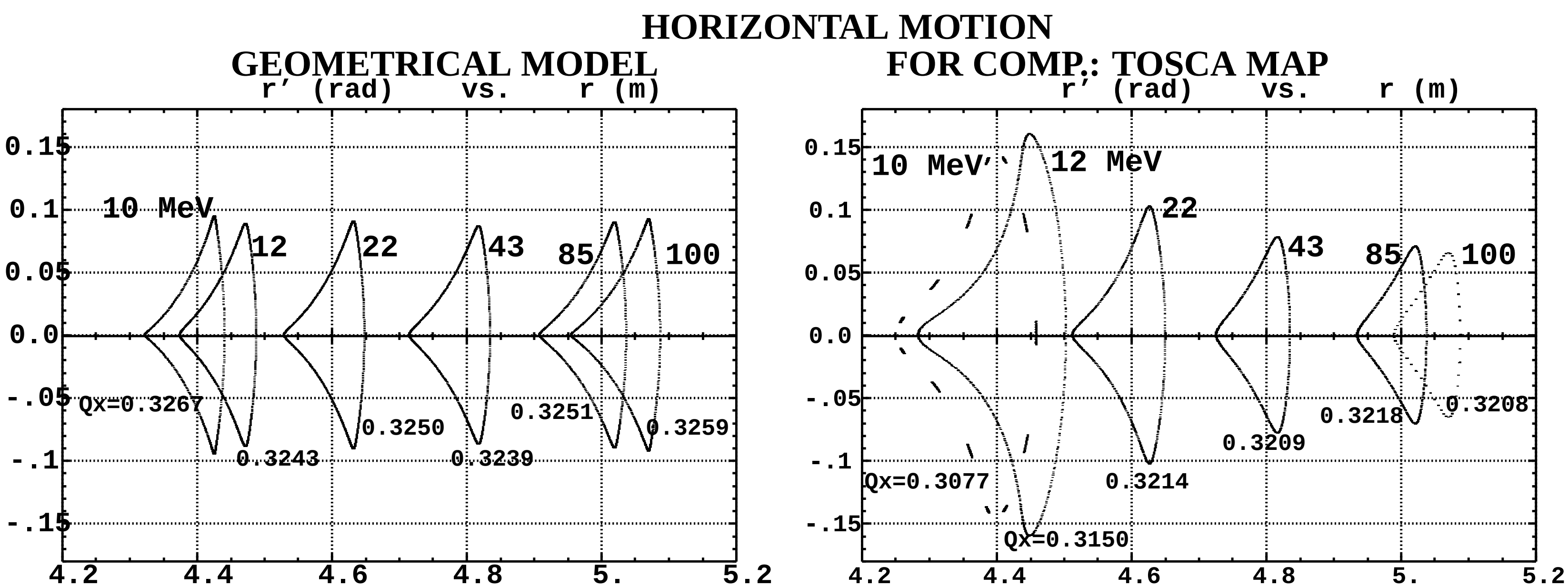


Figure 2: Radial tune (left plot) and axial tune (right) as a function of energy, as obtained using RK4 integration (solid lines/crosses) and using Zgoubi (dashed line/squares).

Stability limits



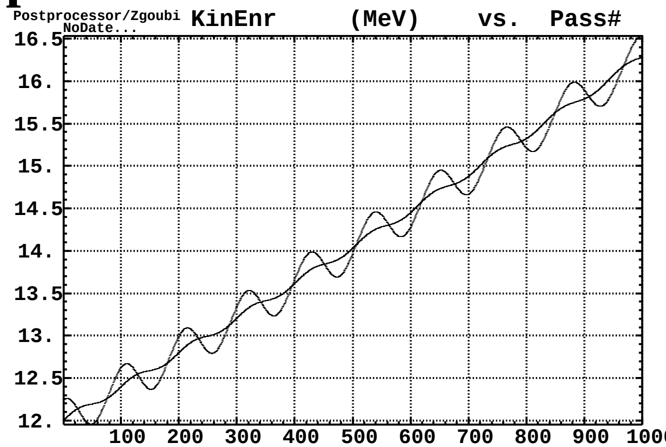
The limits of stable motion for 5 energies, with better than $\Delta r = \pm 0.1$ mm accuracy.

Full acceleration cycle, from 12 to 150 MeV is now experimented.

Characteristics of the acceleration after [KEK Pubs.] :

- $\hat{V} = 19$ **kV,**
- $\phi_s = 20$ **degrees**

RF frequency after previous ray-tracing results : from 1.62 to 4.46 MHz.



Sample synchrotron motion. 2 particles : $r_0 = 4.48$ m and respectively E=12 MeV and 12 MeV+1%.

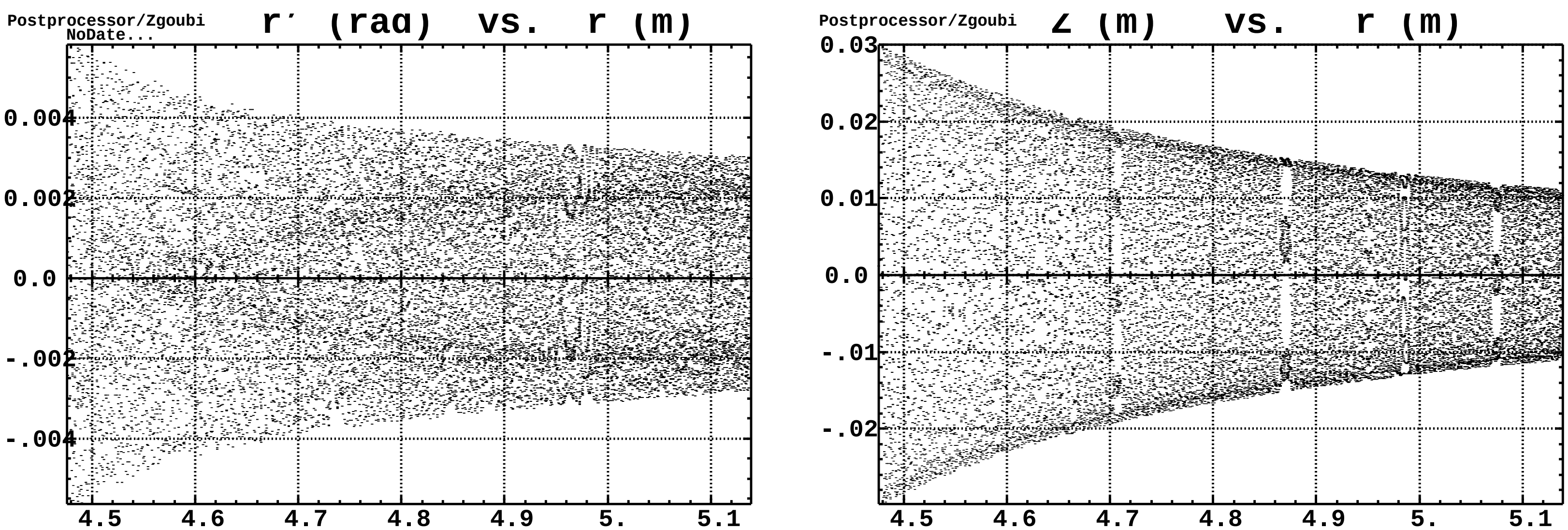


Figure 3: Acceleration from 12 MeV to 150MeV about (from 1.62 to 4.46 MHz), $\phi_s = 20$ degrees, $r_o = r_{c.o.}$(12 MeV), $z_0 = 3$ cm.
Horizontal phase-space (right) and vertical motion (vs. r) ; observation is at center of drift.

Conclusion : ready for 6-D tracking !

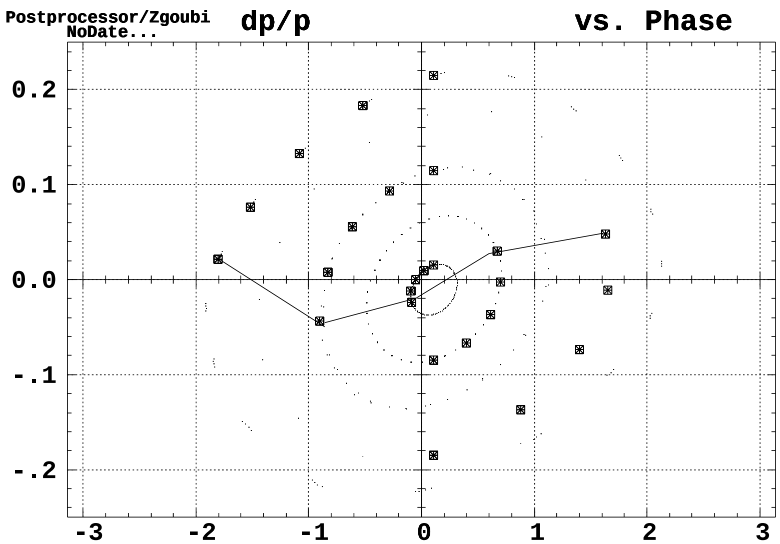
3 Application : Phase rotation

In PRISM, $p_0 = 68$ MeV/c ±20%. E≈ 20 ± 7 MeV

Working hypothesis : the earlier design with 8 cells. Design/geometrical parameters as in publiations, except for : only one cavity,

For simplicity just 5 particles are tracked, representative of the momentum span.

No optimization has been done, just want to show that the geometrical method for 3-D field simulation can be applied starightforwardly, from simply the geometrical parameters of the DFD triplet.



5 first turns (of a 400-turn tracking). The initial beam is the upright one in the dp-phi space, the solid line shows the final beam.

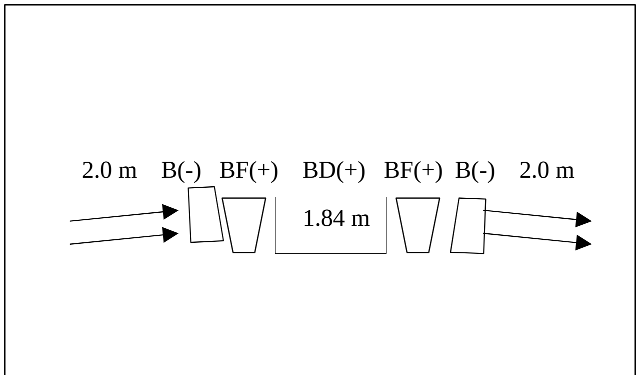
constant gap :			variable gap (α(5/r)^3*6.3cm)	
p/p0	1-Q_x / 1-Qz	beta_x / z	p/p0	1-Q_x / 1-Qz
1.2	0.591252 / 0.665596	0.331 / 1.6141.2	1.2	0.611478 / 0.584989
1.1	0.590575 / 0.673290	0.327 / 1.6011.	1.1	0.611109 / 0.609725
1.	0.589929 / 0.680048	0.322 / 1.5851.	1.	0.610565 / 0.639506
.9	0.589110 / 0.688629	0.317 / 1.569.9	.9	0.609959 / 0.674098
.8	0.588170 / 0.698624	0.311 / 1.551.8	.8	0.609184 / 0.715739

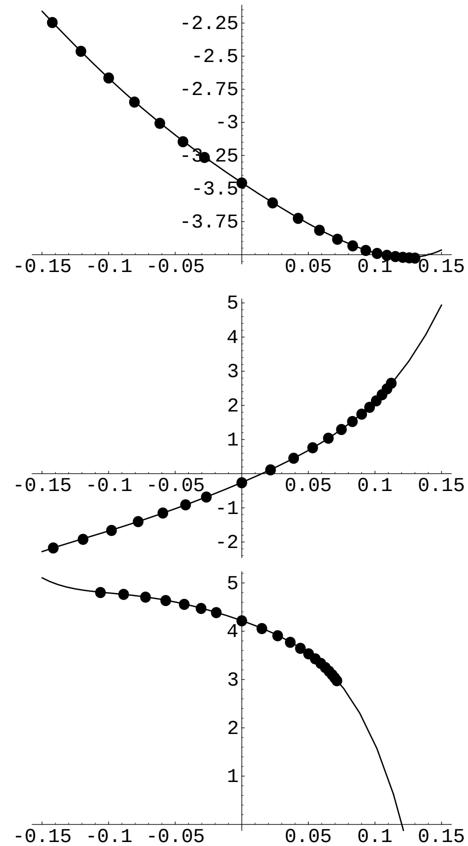
Zgoubi data file, PRISM

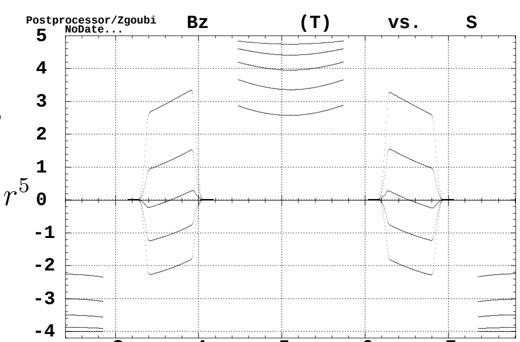

```
  'OBJET'   * c.o., constant Gap *                                                                 1
 226.8235847      68MeV/c muon
 2     5  1
 499.377    0.    0.  0. 0.  1.2     'b'
 492.188    0.    0.  0. 0.  1.1     'a'
 484.4444    0.    0.  0. 0.  1.     'o'
 476.020    0.    0.  0. 0.  .9     'o'
 466.78    0.    0.  0. 0.  .8     'c'
 'FFAG'
 3   45.     500.                        NMAG, AT=tetaF+2tetaD+2Atan(XFF/R0), R0
 ---
 | 18.17   0.  -0.717  5.                    mag 1 : ACNT, dum, B0, K
 | 6.3  0.                                EFB 1 : lambda, gap const/var=0/>0
 |4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 | 1.23 0.    1.E6  -1.E6  1.E6  1.E6
 | 6.3  0.                                EFB 2
D | 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 |-1.23  0.    1.E6  -1.E6  1.E6  1.E6
 | 0. -1                                   EFB 3 : inhibited by iop=0
 | 0  0.      0.      0.      0.      0. 0.  0.
 | 0.  0.   0.    0.    0. 0.
 ---
 | 22.5   0.   3.2    5.                   mag 2 : ACNT0.3927rad, m, B0, K,dummies
 | 6.3  0.                                EFB 1
 | 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 | 3.   0.    1.E6  -1.E6  1.E6  1.E6
F | 6.3  0.                                EFB 2
 | 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 |-3   0.    1.E6  -1.E6  1.E6  1.E6
 | 0. -1                                   EFB 3
 | 0  0.      0.      0.      0.      0. 0.  0.
 | 0.  0.   0.    0.    0. 0.
 ---
 |26.83  0.  -0.717  5.                    mag 3 : ACNT, dum, B0, K
 | 6.3  0.                                EFB 1
 | 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 | 1.23 0.    1.E6  -1.E6  1.E6  1.E6
D | 6.3  0.                                EFB 2
 | 4  .1455   2.2670  -.6395  1.1558  0. 0.  0.
 |-1.23  0.    1.E6  -1.E6  1.E6  1.E6
 | 0. -1                                   EFB 3
 | 0  0.      0.      0.      0.      0. 0.  0.
 | 0.  0.   0.    0.    0. 0.
 ---
 0     2  125.              KIRD anal/num (=0/2,25,4), resol(mesh=step/resol)
 .5                           integration step size (cm)
 2   0.  0. 0. 0.
 --- 8 such FFAG set of data
 'CAVITE'             CAVITY    (only one, 2.2 MV)
 6
 5e6   20.       f0 (Hz),  starting synch Ekin W_s0 (eV)
 2200000.  0.                     Vp (V),  phis (rad)
 'REBELOTE'                                                                           8
 99  0.1 99
 'END'                                                                                9
```

4 Application : 8 to 20 GeV isochronous ring

Magnetic field in bd, BF and BD.
The lower plot shows the field in a cell along the 20, 14, 11, 9.5 and 8 GeV closed orbits.







G. Rees design.
Needs precision tracking : the isochronism has to be controlled at a 10^{-6} precision.

It means accuracy is necessary on
- the description of magnetic field in optical elements,
- ray-tracing.

$$B_{bd}(x) = -3.45623 - 6.689211\,x + 9.403200\,x^2 - 7.623605\,x^3 + 360.3808\,x^4 + 1677.7968\,x^5$$

$$B_{BF}(r) = -0.25776 + 16.62046\,r + 29.73987\,r^2 + 158.65762\,r^3 + 1812.1753\,r^4 + 7669.5302\,r^5$$

$$B_{BD}(x) = 4.22034 - 9.65952\,x - 45.4722\,x^2 - 322.1230\,x^3 - 5364.3096\,x^4 - 27510.421\,x^5$$

Stability limits

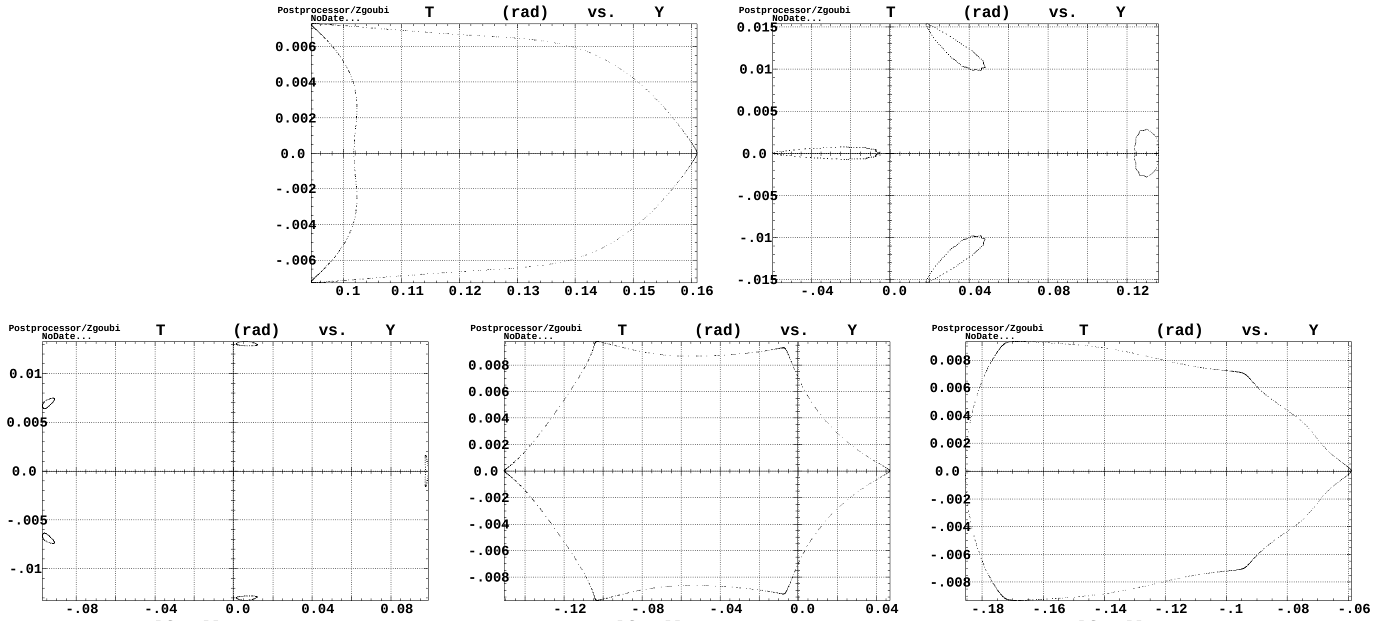


Figure 4: 1000-cell, stability limits of pure horizontal motion, at better than 0.1 cm precision.

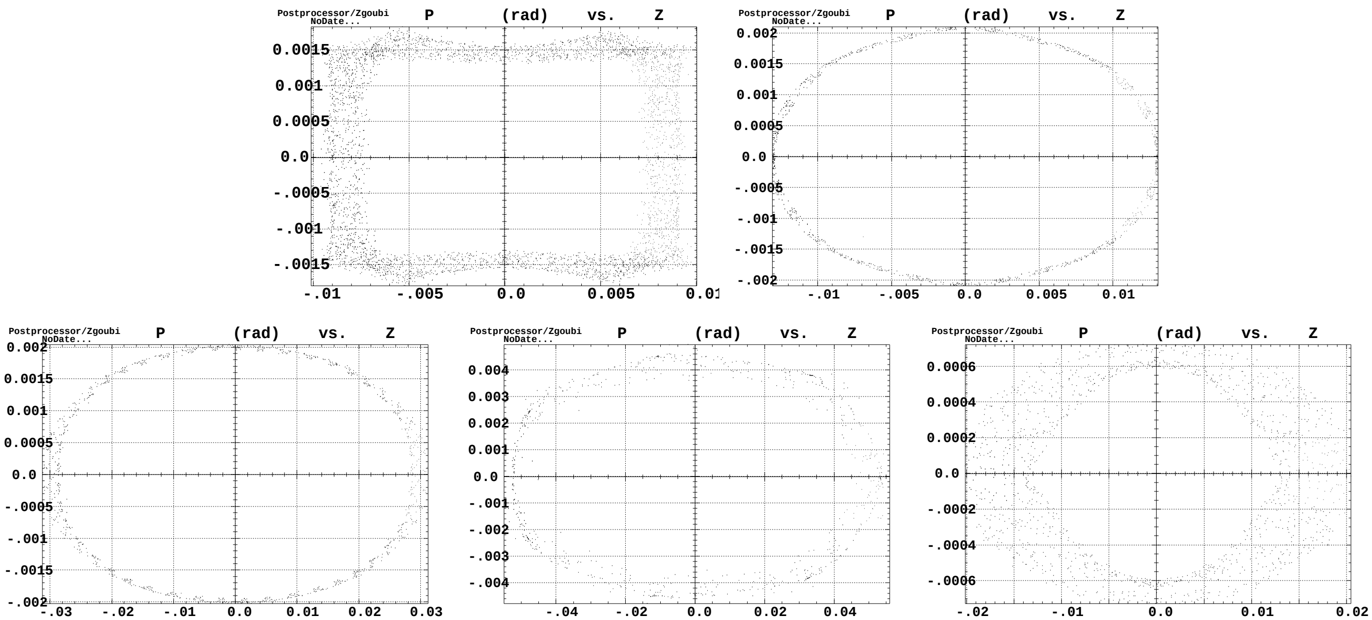


Figure 5: 1000-cell or more, vertical motion stability limits, at better than 0.1 cm precision.

Amplitude detuning

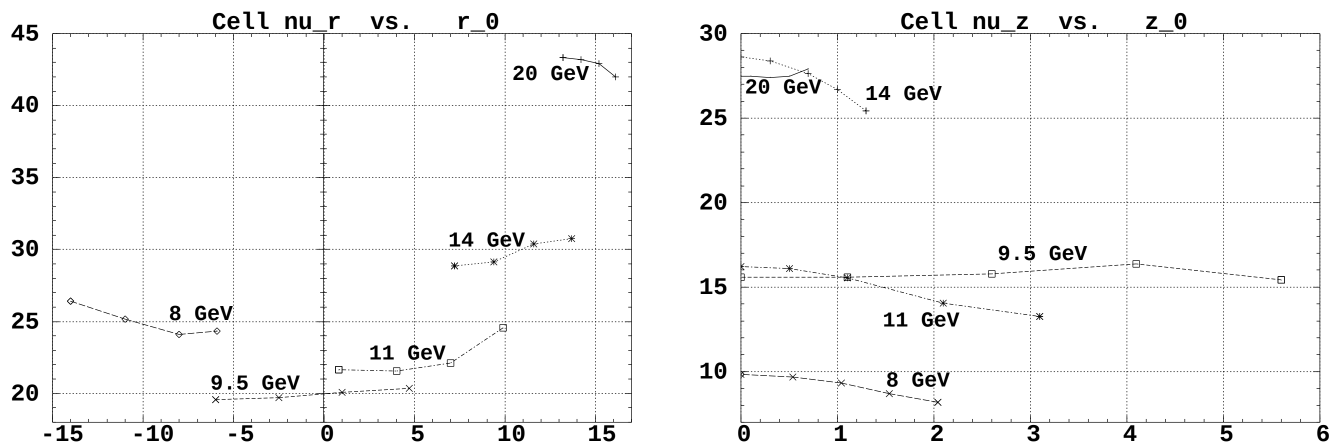


Figure 6: Amplitude detuning. Left : pure radial motion, with for each energy, x_0 varied from closed orbit position to maximum stable amplitude. Right : axial motion, with for each energy, z_0 varied from zero to maximum stable amplitude while $x \equiv x_{c.o.}$ always.